<Contributed Talk 7>NOISE-ENHANCED RESPONSE OF NONLINEAR OSCILLATORS

Perkins, Edmon; Balachandran, B.

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Edmon Perkins and B. Balachandran

Department of Mechanical Engineering
University of Maryland, College Park, MD 20742-3035, USA

Although noise is usually considered undesirable for the operation of a system, for certain nonlinear systems, noise can be of beneficial value. An example of a nonlinear phenomenon that can be beneficial is the phenomenon called *stochastic resonance*, which was first proposed by Benzi, Supera, and Vulpiani to explain the recurrence of ice ages [1].

The authors have an interest in the constructive use of noise and nonlinearity in coupled oscillator systems, and as a first step in this direction, they have explored noise-enhanced responses in the context of the Düffing oscillator by using information theory metrics [2, 3]. After augmenting the equation of motion for the Düffing oscillator with a noise term, the Langevin equation can be obtained as [4]

$$\mathrm{m}\,\mathrm{d}(\dot{x}) = (-\mathrm{a}\,x - \mathrm{b}\,x^3 + \mathrm{e}\cos\omega t)\mathrm{d}t - \mathrm{c}\,\mathrm{d}x + \sqrt{2\mathrm{D}}\,\mathrm{d}W. \tag{1}$$

After using an adiabatic elimination for simplification [5], the Euler-Maruyama Method [6] was used to numerically solve the stochastic differential system. After considering negative-valued responses to be zeros and positive-valued responses to be ones, the information rate was calculated, using Shannon entropy and conditional entropy [2]. The findings presented in Figures 1 to 3 suggest that appropriate noise levels can be determined to have response amplification.

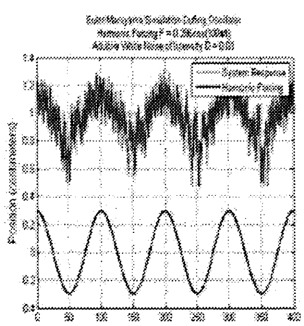

Figure 1: With a low level of noise, the oscillator is trapped in one well [3].

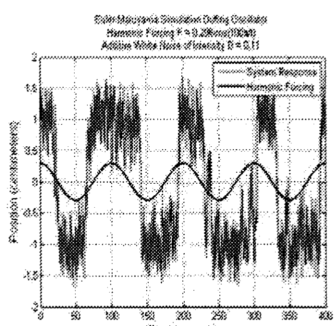

Figure 2: With an ideal amount of noise, the oscillator response is enhanced [3].

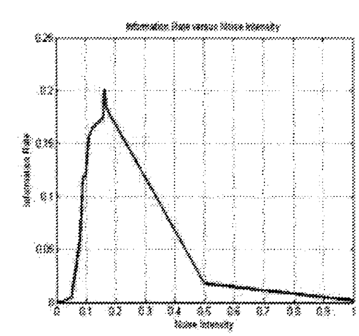

Figure 3: There is a peak of mutual information when the noise intensity is near 0.15 [3].

As an example of the coupled oscillator system to be studied by the authors, Figure 4 is provided. Building on earlier work conducted [8, 9], the authors plan to study the formation of intrinsic localized modes (ILMs) in these coupled oscillator systems and the effect of noise on them. Information theory metrics will be used to measure the benefit of the noise. An enhanced understanding of the dynamics of the coupled oscillator array in the presence of noise can provide means to control ILMs.

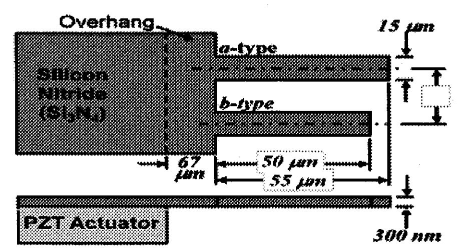

Figure 4: Unit cell of array, as considered by Sato *et al.* [7].

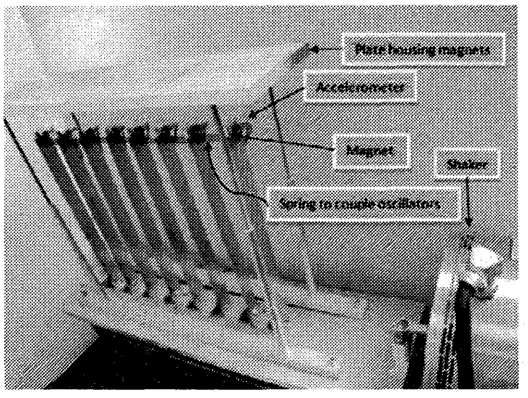

Figure 5: Experimentally constructed macro-array of coupled oscillators.

The experimental arrangement shown in Figure 5 will be used in further studies, and the findings will be included in the full article.

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